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SS 2002
1. 5. 2002

Lösungen zum 2. Übungsblatt zur Mafi II

Lösung zu Aufgabe 5:

(a)

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 2 & -3 & 0 & 3 \\ -4 & 13 & 6 & -7 \end{array}\right) \overset{II-2I, III+4I}{\Leftrightarrow} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -7 & -6 & 1 \\ 0 & 21 & 18 & -3 \end{array}\right) \overset{III+3II}{\Leftrightarrow} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -7 & -6 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\begin{aligned} \Rightarrow \quad & x_3 = \lambda \\ & x_2 = -\tfrac{6}{7}\lambda - \tfrac{1}{7} \\ & x_1 = -2\left(-\tfrac{6}{7}\lambda - \tfrac{1}{7}\right) - 3\lambda + 1 = -\tfrac{9}{7}x_3 + \tfrac{9}{7} \end{aligned}$$

$$L = \left\{ \left(-\frac{9}{7}\lambda + \frac{9}{7}, -\frac{6}{7}\lambda - \frac{1}{7}, \lambda \right) | \lambda \in \mathbb{R} \right\}$$

(b)

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 1 & 2 \\ 0 & -1 & 4 & -1 & -1 \\ 4 & -1 & 1 & -1 & 0 \end{array}\right) \overset{III-2I}{\Leftrightarrow} \left(\begin{array}{cccc|c} 2 & 0 & -1 & 1 & 2 \\ 0 & -1 & 4 & -1 & -1 \\ 0 & -1 & 3 & -3 & -4 \end{array}\right) \overset{III-II}{\Leftrightarrow} \left(\begin{array}{cccc|c} 2 & 0 & -1 & 1 & 2 \\ 0 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & -2 & -3 \end{array}\right)$$

$$\begin{aligned} \Rightarrow \quad & x_4 = \lambda \\ & x_3 = -2\lambda + 3 \\ & x_2 = -8\lambda + 12 - \lambda + 1 = -9\lambda + 13 \\ & x_1 = -\lambda + \tfrac{3}{2} - \tfrac{1}{2}\lambda + 1 = -\tfrac{3}{2}\lambda + \tfrac{5}{2} \end{aligned}$$

$$L = \left\{ \left(-\frac{3}{2}\lambda + \frac{5}{2}, -9\lambda + 13, -2\lambda + 3, \lambda \right) | \lambda \in \mathbb{R} \right\}$$

(c)

$$\left(\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 3 \\ -2 & 1 & -6 & 1 & 2 \\ 5 & -3 & 13 & -2 & 5 \end{array}\right) \overset{II+2I, III-5I}{\Leftrightarrow} \left(\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 3 \\ 0 & -1 & -4 & 1 & 8 \\ 0 & 2 & 8 & -2 & -14 \end{array}\right) \overset{III+2II}{\Leftrightarrow} \left(\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 3 \\ 0 & -1 & -4 & 1 & 8 \\ 0 & 0 & 0 & 0 & 6 \end{array}\right)$$

$$\Rightarrow L = \{\emptyset\}$$

Lösung zu Aufgabe 6:

$$\left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 2 & 9 & 1 & 5 \\ 1 & 5\alpha+2 & 2 & 3+\alpha \end{array}\right) \overset{II-2I, III-I}{\Leftrightarrow} \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & 2\alpha+1 & 3 \\ 0 & 5\alpha & \alpha+2 & \alpha+2 \end{array}\right)$$

Wir unterscheiden abhängig von α folgende Fälle.

- Fall 1: $\alpha = 1$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & 3 & 3 \\ 0 & 5 & 3 & 3 \end{array}\right)$$

$$\begin{aligned} \Rightarrow \quad & x_3 = \lambda \\ & x_2 = \tfrac{3-3\lambda}{5} \\ & x_1 = 1 + \lambda - 2\left(\tfrac{3-3\lambda}{5}\right) = \tfrac{11}{5} - \tfrac{1}{5}\lambda \end{aligned}$$

$$L = \left\{ \left(-\frac{1}{5}\lambda + \frac{11}{5}, \frac{3-3\lambda}{5}, \lambda\right) |\lambda \in \mathbb{R} \right\}$$

- Fall 2: $\alpha = -1$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & -1 & 3 \\ 0 & -5 & 1 & 1 \end{array}\right) \overset{-1\cdot III}{\Leftrightarrow} \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & -1 & 3 \\ 0 & 5 & -1 & -1 \end{array}\right) \overset{III-II}{\Leftrightarrow} \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & -1 & 3 \\ 0 & 0 & 0 & -4 \end{array}\right) \Rightarrow L = \{\emptyset\}$$

- Fall 3: $\alpha \in \mathbb{R} \setminus \{-1, 1\}$

$$\overset{III-\alpha\cdot II}{\Rightarrow} \left(\begin{array}{ccc|c} 1 & 2 & -\alpha & 1 \\ 0 & 5 & 2\alpha+1 & 3 \\ 0 & 0 & \alpha+2-\alpha(2\alpha+1) & \alpha+2-3\alpha \end{array}\right)$$

$$\begin{aligned} \Rightarrow \quad & x_3 = \tfrac{-2\alpha+2}{-2\alpha^2+2} = \tfrac{1}{1+\alpha} \\ & x_2 = \tfrac{1}{5}\left(3 - (2\alpha+1)\tfrac{1}{1+\alpha}\right) = \tfrac{2+\alpha}{5+5\alpha} \\ & x_1 = 1 + \tfrac{\alpha}{1+\alpha} - 2\left(\tfrac{2+\alpha}{5+5\alpha}\right) = \tfrac{1+8\alpha}{5+5\alpha} \end{aligned}$$

Für ein gegebenes α existiert genau eine Lösungsmenge, nämlich:

$$L = \left\{ \left(\frac{1+8\alpha}{5+5\alpha}, \frac{2+\alpha}{5+5\alpha}, \frac{1}{1+\alpha}\right) \right\}$$

Lösung zu Aufgabe 7:

(i) E_1:

$$\left(\begin{array}{cccc|c} 2 & 4 & -1 & -1 & 0 \\ 1 & -4 & 2 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 \end{array}\right) \overset{II\Leftrightarrow III}{\Rightarrow} \left(\begin{array}{cccc|c} 2 & 4 & -1 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 1 & -4 & 2 & -1 & 0 \end{array}\right) \overset{III-\frac{1}{2}I}{\Rightarrow} \left(\begin{array}{cccc|c} 2 & 4 & -1 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & -6 & 2,5 & -0,5 & 0 \end{array}\right)$$

$$\overset{III+3II}{\Rightarrow} \left(\begin{array}{cccc|c} 2 & 4 & -1 & -1 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 0 & 5,5 & -3,5 & 0 \end{array}\right)$$

d wird als 11 festgesetzt:

$$\begin{aligned} d &= 11 \\ c &= 7 \\ b &= 2 \\ a &= 5 \end{aligned}$$

$$\Rightarrow E_1 := \left\{(x,y,z) \in R^3 | (5x + 2y + 7z = 11)\right\}$$

E_2:

$$\left(\begin{array}{cccc|c} 0 & 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & -1 & 0 \end{array}\right) \stackrel{sort.}{\Rightarrow} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 2 & 0 & 3 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right) \stackrel{II-2I}{\Rightarrow} \left(\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 0 \\ 0 & -2 & 3 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{array}\right)$$

d muß 0 sein, c ist beliebig:

$$\begin{aligned} d &= 0 \\ c &= 2 \\ b &= 1 \\ a &= -3 \end{aligned}$$

$$\Rightarrow E_2 := \left\{(x,y,z) \in R^3 | (-3x + y + 2z = 0)\right\}$$

(ii)

$$\begin{aligned} 5x + 2y + 7z &= 11 \quad I \\ -3x + y + 2z &= 0 \quad II \\ +\frac{11}{5}y + \frac{31}{5}z &= \frac{33}{5} \quad I + \frac{5}{3}II \end{aligned}$$

z wird ersetzt durch frei wählbares λ:

$$\begin{aligned} z &= \lambda \\ y &= \frac{2}{11}\lambda \\ x &= \frac{8}{11}\lambda \end{aligned}$$

$$G_1 := \left\{(x,y,z) \in R^3, \lambda \in \mathbb{R} | x = \frac{8}{11}\lambda, y = \frac{2}{11}\lambda, z = \lambda\right\}$$